Developing DECIMAL Sense

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describes a sequence of activities to help develop students' understandings of decimal fractions.

Background issues

I recently read a research paper by Padberg (2002), in which the development of understanding associated with decimal fractions was studied. Padberg (2002) outlined the situation that existed in Germany, where students were introduced to decimal fractions in the sixth year of school. He claimed that it was assumed students would have a deep understanding of decimals because of their prior knowledge of decimals through real life associations. Germany, like Australia, operates around a decimal system, and students are frequently seeing and working with decimals in everyday situations.

When decimals were introduced into the classroom, Padberg (2002) noted that it was assumed the students would easily make the transition from this real life experience to abstract notation and demonstrate a deep understanding of the concept of decimals. The results of the research, however, indicated that students did not easily make this transition from the real life concrete experiences to abstract notation used in the classroom.

In Australia, there exists a similar context in which students are introduced to decimals in the fourth and fifth years of schooling. As with the situation in Germany, our students are also familiar with using decimals in real life with decimal notation found in everything from money to petrol prices, displayed prominently in a variety of forms and discussed frequently in conversation both at home and at school.

My experience as a classroom teacher in the middle years of schooling supports the conclusions of Padberg, that despite the evidence of decimals in real life and the frequent use of decimals

in written and spoken form, students do not translate this background knowledge across to abstract contexts in the classroom. Thus, I began developing the following sequential approach to understanding decimals which has achieved remarkable success in student understanding and their processing of decimal fractions.

Beginning with decimals and concrete materials

Initially, I introduce students to the concept of decimals as fractions; parts of a whole. Students are presented with a lump of playdough and asked to divide this playdough into two equal amounts.

One section is placed to the side and the second section is then divided into tenths, or ten equal parts (see Figure 1).

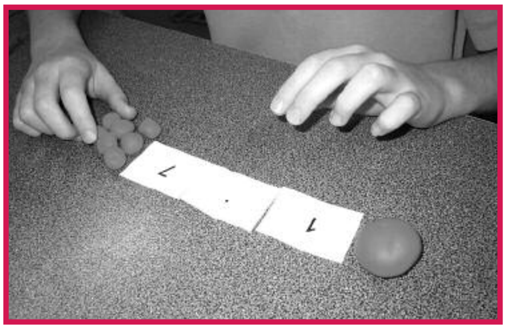

Figure 1: Dividing the playdough

I then take the whole section and compare it to the tenths, asking "Do these tenths still equal the same amount as the whole piece of playdough?" Not surprisingly, many students are still forming the concepts of parts of a whole. If students do not have this concept of conservation of number firmly established, the understanding of the written form of decimals will have very little foundational support.

When students understand the concept of decimals as parts of a whole number, I introduce written notation using cards. Students use the cards to create the concrete representations of the whole numbers and decimals using the playdough (see Figure 2). At this time, I also begin to establish the language of decimal notation as "tenths" and "hundredths" rather than "point" or "decimal". So 3.2 is read as "three and two tenths" rather than "three point two" or "three decimal two". By establishing the language to match the concrete representations and the written notation, a more accurate association between the three elements is established.

After the initial foundation activities, I begin to challenge students' understanding by presenting written notation that looks similar but represents different decimal values. By using the playdough materials, students are able to create the decimal values using concrete materials to match the written notation on the cards.

Extending to hundredths and thousandths

Very soon, I challenge students to predict and describe how hundredths could be formed using playdough (see Figure 2).

The students usually begin with tenths and extrapolate to predict that if each tenth were further subdivided into ten smaller parts, this would produce one hundred smaller parts of the whole piece of playdough. Students become engaged in the laborious process of physically creating hundredths (see Figure 3), but the time and energy spent on this activity is irreplaceable in terms of conceptual development.

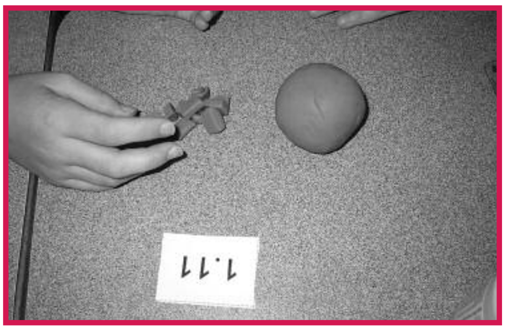

Figure 2: Using the playdough and cards to make decimal numbers

Most amazing is that the students usually want to then predict how to create thousandths and the discussions, methods and results have been a revelation at times of student creativity and innovative mathematical processing. I have seen students using metre rulers and measuring long strips of playdough into millimetres.

I have also seen students work together in small groups with calculators to discover if a thousand is a square number and can be created through a grid pattern rather than individual pieces. The conceptual memories that are embedded through these activities engage several mathematical processes, not just knowledge of decimals.

Figure 3: Making hundredths

Zero values and ordering values

As students work with concrete representations of whole numbers, tenths and hundredths, numerical cards are frequently employed to challenge and engage their thinking and clarify understandings of decimals. The zero value is introduced at this point and students discuss and represent numbers that look similar but represent different values because of the placement of the zero (see Figure 4).

Figure 4

After establishing the impact of zero in a decimal number, I often use only numerical cards and ask students to order the cards to demonstrate their understanding and abilities to work with the abstract forms of decimal notation. Students are also consistently engaging with the oral language of decimals and read written notation using the terms tenths and hundredths. I also engage students in using blank number lines to represent both whole numbers and decimals. This develops the concept of decimals as numbers that are situated between whole numbers. The decimals that begin with a zero present the greatest challenge as students often have difficulty perceiving that there are numbers between 0 and 1.

Links to decimals in real life

This is also the time when I begin to introduce a challenge by linking decimals in real life to the abstract concepts that the students have been developing. As decimals in real life are often presented in the form of money, measures or values such as petrol prices, most of the time the language of everyday use is different to the decimal language used in the classroom. By confronting this problem and raising students' awareness of the use of decimals and the variations in the language used; most students are able to see and recognise the decimal fraction as both an everyday term and a mathematical value.

Once these links are understood, it is a natural transition to begin using decimals in

problem solving and computations. Processing computations using decimals is usually not a difficult task if the foundation has been established of understanding what decimals are and how they are written. The place value of decimals relies on more than knowledge of columns and decimal points.

Making connections across different areas

Making connections or associations across strands and concepts in mathematics enhances understanding and builds higher levels of knowledge and application skills (NCTM, 2000).

As students work with decimals, the first links are made between decimals and common fractions. This connection occurs very early with the use of concrete materials to create tenths. As well as writing these representations using decimal notation, I also use the common fraction notation for tenths to build this association. As students are introduced to other concepts such as percentage, measurement conversions, reading scales and using money values, the opportunities to reinforce the connections are limitless. I constantly endeavour to raise students' awareness of the connections that exist between decimals and other areas of mathematics. The need to draw on knowledge of several concepts in most activities is constantly reinforced.

Conclusion

It has been my experience that this sequence of conceptual development requires substantial time for students to play with decimals and create decimals using concrete materials. Substantial time may include immersion activities ranging over twenty to twenty-five hours of mathematics. My mathematics activities are conducted in blocks of approximately an hour and usually focus on one particular concept across a week. So in a six month period I may engage students in these concept building activities over four or five weeks (not consecutively). Once the concept of decimal values and notation is formed through these concrete experiences, students appear to have very little difficulty moving to abstract notation and processes. The sense of decimals, both as a mathematical process and as a natural part of our lives is much more grounded in meaningful experiences for the students. Whereas in the past, time was spent learning the mathematical processes involving decimals and repeatedly returning to place value, the concrete methods appear to support abstract understandings far more comprehensively rendering repetition pointless.

References

Padberg, F. (2000). Transition from concrete to abstract decimal fractions. In B. Barton, K. Irwin, M Pfannkuch & M. Thomas (Eds), *Mathematics Education in the South Pacific: Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia* (pp.536–542), Sydney: MERGA.

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Rosemaree Caswell has worked in education for over twenty years with experience as a classroom teacher, education advisor and consultant. She completed a Master of Education Studies in 2005 researching aspects of mathematics learning. She is passionate about engaging students effectively in mathematics learning and making learning mathematics fun, meaningful and lasting.
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